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Dirac- and Lorentz-Invariant Symmetries in Mass and Four-Momentum for Superluminal Aspects

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Abstract

Superluminal concepts since 1962 have continued to gain momentum for numerous discussions. The so-called (yet unconfirmed) tachyons have been basically dealt in the literature with a second-order field equation, leaving aside a probable exploitation through a simpler first-order mechanism. Initiating this (first-order field) approach we show that the Dirac invariance under joint reversal symmetries of the mass and four-momentum leads to a (generalised) Lorentz symmetry. This yields solutions of a faster-than-light particle of definite negative mass (referred to here as 'bisiston') with possibly annihilating and other features. How can the 'minus' sign in the mass, $-m$, be interpreted? It seems, it corresponds to a mass-repulsion process leading to a very probable form of the (missing) ± symmetry in the mass (i.e. gravitational) interaction (comparable to the universal $V \pm A$ result). This argument appears to be extremely plausible in context of our wide universe where such a symmetry cannot be ruled out in the large.

1. *Introduction*

For many years symmetry principles have played a fundamental role in developing new physical ideas. Heat usually flows from hot to cold regions, but the symmetrical aspect provides for its direction to be reversed. In fact, we have built sophisticated mechanisms (such as refrigerators) where heat appears to flow from cooler to warmer bodies. Similarly, the notion of symmetry in time (i.e., time-reversal) has contributed greatly to developments of the propagator's approach (e.g., Feynman, 1962). And the proposition of the hole theory (Dirac, 1930) gives rise to a realisable symmetry for particles and anti-particles in nature. Likewise, the positive and negative characters of electrical charges and magnetic poles conform superbly to symmetrical concepts corresponding to attractive and repulsive features for the electromagnetic interaction. The strong interaction as well follows a similar trend, e.g., both the features of attraction and (hard-core) repulsion in nuclei are known. One may now ask why such symmetrical features should lack in the mass (gravitational) interaction, G_I. We may go further and add whether it is possible to develop some complex ideas which would permit us to include both the attractive and repulsive features for the G_I (gravitational interaction). Plainly speaking, it would mean that the product of two interacting masses (characterising the gravitational force in between) need to be symmetric,* i.e. +ve and −ve both. In order to be so, at least any of the two masses must be of opposite sign to each other. Is there anything in principle forbidding this condition? Certainly not, if the Dirac theory can provide conditions to conform to this aspect. In fact, the mass-reversal (i.e., $m \to -m$) invariance of the Dirac equation has been shown by Sakurai (1958) (see also Roman's textbook, 1960) to lead to a universal $V \pm A$ interaction—a result which is also obtained by Sudershan & Marshak (1958) from a chirality transformation. However, nowhere in Sakurai's formalism in explicit use made of $m \to -m$ (Roman, 1960), so that further probable implications, such as the $\pm\, G_I$, possibly become suppressed.

It is the purpose of this paper to make an attempt to understand this point by pursuing the invariance of the Dirac equation under joint reversals of the four-momentum and mass of a particle, e.g., the electron of mass m and charge 'e'. We find that the Dirac-invariant conditions in our formalism give the Lorentz-invariance symmetry. This leads to solutions of a faster-than-light particle of charge e and mass $m' = -m$ with properties yet to be observed or discovered. For example, a m'-particle is likely to interact with its pair-electron via two distinct processes, annihilation and mass (gravitational) repulsion, as we shall now see.† Hereafter, for the sake of convenience, we shall refer to a negative-mass-in-effect ($m' = -m$) particle as 'bisiston'.‡

2. *Theoretical Formulation, Results and Discussions*

We start with the familiar Dirac equation for the electron in the Feynman slash notation (see Bjorken & Drell, 1964) as follows:

$$(\not{p} - e\not{A} - m)\psi^{\omega} = 0 \qquad (\omega = 1, 2, 3, 4) \tag{2.1}$$

* Ordinarily, the gravitational force between masses m' and m is given by $(Gm'm/R^2)$, where G is the gravitational constant and the distance-squared quantity R^2 is always +ve. The sign of the product $(m'm)$, therefore, characterises the field. In notation, where a +ve sign corresponds to the mass-attraction, a −ve sign would mean the mass-repulsion. The ± symmetry in G_I refers to this aspect.

† Throughout, we use schemes of notations given in the text of Bjorken & Drell (1964): $\hbar = c = 1$; the contravariant four-vector such as the space-time coordinates $x^{\nu} (\equiv (x^0, x^1, x^2, x^3) \equiv (t, x, y, z)$ is related to its covariant quantity $x_u (\equiv (t, -x_i) \equiv (t, -x_1, -x_2, -x_3))$ as $x_{\mu} = g_{\mu\nu} x^{\nu}$ with $g_{\mu\nu} = 0$ $(\mu \neq \nu)$, $g_{00} = -g_{ii} = 1$ $(i = 1, 2, 3)$, $x_{\mu} x^{\mu} = t^2 - x_i^2$, $g^{\mu\nu} = g_{\mu\nu}$, $(g_{\mu\nu})^2 = I$, a unit 4 x 4 matrix. Notice that non-vanishing elements of $g_{\mu\nu}$ can be ±1, and $(-g_{\mu\nu})^2 = I$.

‡ From a Sanskrit word meaning a particle of exclusively distinct character. We might also call it a 'negative (mass) tachyon'. Instead of a second-order field equation as used for tachyons (Dhar & Sudarshan, 1968) we initiate a first-order approach in this paper.

with $\not{p} = \gamma^\mu p_\mu = i\not{\nabla}$, $\nabla_\mu = (\partial/\partial x^\mu)$ ($\mu = 0, 1, 2, 3$); $\gamma^0 = \beta$, $\gamma^i = \beta\alpha_i$ ($i = 1, 2, 3$); $\not{A} = \gamma^\mu A_\mu$, $A_\mu = g_{\mu\nu} A^\nu$, $p^\mu = g^{\mu\nu} p_\nu$ ($\nu = 0, 1, 2, 3$); and $p^2 = p^\mu p_\mu = E^2 - p_i^2 = m^2$. In the above, E, p, A, refer to the total energy, four-momentum, and electromagnetic four-potential, respectively; β and α_i are the usual Dirac 4 x 4 matrices; and ψ^ω is a 4-component wave function with large ($\omega = 1, 2$) and small ($\omega = 3, 4$) components corresponding to the positive and negative energy, respectively. (i is an imaginary unit.) Clearly, the (Dirac) equation (2.1) is invariant under the following operations:

(I) Multiply (2.1) from the left with a non-singular 4 x 4 matrix, D, as

$$D = \gamma^5 D_o D_o, \qquad (D_o = \gamma^1\gamma^2\gamma^3, \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5) \tag{2.2}$$

(II) and simultaneously reverse the four-momentum and mass via† $g_{\mu\nu} \to -g_{\mu\nu}$, i.e.,

$$p_\mu \to -p'_\mu, \qquad m \to -m', \qquad g_{\mu\nu} \to -g_{\mu\nu}$$

implying further (2.3)

$$A_\mu \to -A'_\mu, \qquad p^\mu (= g^{\mu\nu} p_\nu) \to +p'^\mu$$

(Here it is easy to misunderstand these steps to be trivial, but deeper insight is required to exploit their real significance, as we shall soon see. Note also that to lowest order D is just equal to γ^5, and higher orders of D are of no importance.) From the usual anti-commutator relations of γ-matrices (Bjorken & Drell, 1964) we verify the following:

$$Dm' - m'D = 0, \qquad D\gamma^\mu + \gamma^\mu D = 0 \tag{2.4}$$

$$DQ + QD = 0 \qquad (Q = \not{p}, \not{A}, \not{S}) \tag{2.5}$$

$$D\psi^\omega = \psi'^\lambda, \qquad \lambda(= 1, 2, 3, 4) \to \omega(= 3, 4, 1, 2) \tag{2.6}$$

$$(\not{p}' - e\not{A}' + m')\,\psi'^\lambda = 0 \tag{2.7}$$

This invariance, obtained through equations (2.2) and (2.3), amounts to saying that equation (2.7) is just as good for the m' ($= -m$)-particle (bisiston) as is the original Dirac equation for the m-electron, and the two solutions ψ^ω and ψ'^λ are related to each other by the simple interchange of positive- and negative-energy components where the spin (S_i) does not reverse (as it does for the positron case) but the polarisation (S_μ) does. To see this, let the spin-up negative-energy electron have the solution $\psi^3 = (2\pi)^{-3/2} \exp(imt)$ {0 0 1 0} (Bjorken, 1964) where the column matrix denoted by { } is expanded fully in the following equation. The corresponding solution for the bisiston is then given from above as

† This step automatically yields the four-momentum reversal as well as the Lorentz-invariance symmetry. Otherwise, no significant physics is likely to come out. For a Lorentz-invariance (space-time) symmetry, see other works, e.g., Recami & Mignani, 1972.

$$\psi'^{\lambda} = \gamma^5 D_0 D_0 \psi^3 = (2\pi)^{-3/2} \exp(-im't) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{2.8}$$

$$= (2\pi)^{-3/2} \exp(-im't) \{1 \;\; 0 \;\; 0 \;\; 0\} = \psi'^1(m' = -m) \tag{2.9}$$

In general, using the energy- and spin-projection operators (Bjorken *et al.*, 1964) as $((\epsilon \not{p} + m)/2m)$ and $((1 + \gamma_5 \not{S})/2)$, respectively, with $\epsilon = \pm 1$, $\not{S} = \gamma^\mu S_\mu$, $S^\mu S_\mu = -1$, $S^\mu = (O, S_i)$, $S^\mu p_\mu = 0$, we apply the same transformation (2.6) to an arbitrary spin-momentum eigenstate to obtain

$$\begin{aligned} \psi'^{\lambda} &= D\psi^{\omega} = [\gamma^5 D_o D_o \, ((\epsilon \not{p} + m)/2m) \, ((1 + \gamma_5 \not{S})/2)] \, \psi^{\omega} \\ &= ((+ \epsilon \not{p}' + m')/2m') \, ((1 - \gamma_5 \not{S})/2) \psi'^{\lambda} \end{aligned} \tag{2.10}$$

That is, similar to the case of positrons, equation (2.6) projects a negative-energy eigenstate of any spin-momentum into a positive-energy eigenstate of the same spin-momentum. The sign change in polarisation S_μ is just as formal as ever in such cases indicating that the ∓ 1 eigenvalues of the negative-energy electron's eigenfunctions σ_3 (z-component of the Pauli spin 2×2 matrix) correspond to the ± 1 with respect to the positive-energy bisiston.

Because of the magnetic moment dependence on 'e/m', we see that such a moment for the bisiston ($m' = -m$) is exactly identical with that of the positron (with its charge $e_p = -e$), and satisfies the anti-particle criterion with respect to an electron. In addition, the m' and m system is gravitationally repulsive (-ve sign) as opposed to the attraction (+ve sign) between masses m and m. This, however, leads to $\pm G_I$ (gravitational-interaction) symmetry more or less similar, and/or equivalent, to the universal $(V \pm A)$ interaction result of Sakurai (1958) and Roman (1960), although our approach here is quite different from theirs. Thus we have ψ and ψ' describing the two *identical* physical inertial systems corresponding to N and N', respectively. Each is a complete world in itself containing exclusively, as experience shows, those particles which attract one another gravitationally. The knowledge as to which of the worlds (N or N') we belong is, of course, immaterial or irrelevant. The important aspect now consists of determining the relative behaviour of N with respect to N' through the use of transformation (2.3). We must recourse to two guide lines: (1) The mass m must be a rest-mass in N, while m' must be at rest with respect to N'. This ensures the validity of equations (2.1) and (2.7) individually in the respective systems. Up to this point the sign of mass does not matter at all (see also, Roman, 1960). (2) The invariance requirement given mainly by equation (2.3) must be understood in terms of physical implications. We can also exploit the physical meaning of $m' = -m$ from special relativity, since the two masses here are not *identically* equal. The point we know precisely from the Einstein theory is that two identically equal masses must be at rest with respect to each other.

Since $p_\mu \to -p'_\mu$, and $p^\mu \to +p'^\mu$ in equation (2.3), the energy-momentum relations for the inertial systems N and N' are given as

$$(p^2 =)\, p^\mu p_\mu \to -p'^\mu p'_\mu, \qquad E^2 - p_i{}^2 = -(E'^2 - p_i'^2) \tag{2.11}$$

The physical meaning is now more clear. The transformation (2.3) transforms two-sheeted hyperboloids $p^2 > 0$ into single-sheeted hyperboloids $p^2 < 0$ implying that a m'-particle belonging to N' is *superluminal* with respect to an m-electron of the N-system. (For details regarding superluminal (faster-than-light) transformations and particles, the so-called tachyons, see papers by a host of authors such as Bilaniuk & Sudarshan *et al.* (1962), Feinberg (1967), Parker (1969), Antippa (1972), Recami *et al.* (1972), Sinha (1973), and others. In particular, Mignani & Recami (1973) contains a good bibliography on tachyons.) Similarly, in the homogeneous four-coordinate space-time with isotropy of 3-component (i) equations (2.3) and (2.11) reduce to

$$x^\mu x_\mu \to -x'^\mu x'_\mu, \qquad t^2 - x_i{}^2 = -(t'^2 - x_i'^2) \tag{2.12}$$

which means that space(time)-like intervals belonging to N become time-(space)-like intervals with respect to N'. Here it is comforting to note that such a result, which always constitutes an axiomatic assumption in the tachyon theory (see above-mentioned references), comes directly from conditions (2.2) and (2.3) required in our formalism for the invariancy of the Dirac equation. To see things in a more familiar form, from now on we write results in explicit units of c, the velocity of light in vacuum. The relation (2.12) with $t(t') \to ct(ct')$ gives coordinates transformation as

$$\left.\begin{aligned} &x'_1 = r(x_1 - vt), \qquad t' = r(t - (vx_1/c^2)), \qquad x'_2 = \pm i x_2, \qquad x'_3 = \pm i x_3 \\ &\text{with} \\ &r = \pm\,[(v^2/c^2) - 1]^{-1/2} = \pm i[1 - (v^2/c^2)]^{-1/2} = \pm\,[|1 - (v^2/c^2)|]^{-1/2} \end{aligned}\right\} \quad (v > c) \tag{2.13}$$

which are identical with the 'generalised Lorentz transformation (GLT)' (Parker, 1969; Antippa, 1972; Mignani & Recami, 1973) for superluminal (inertial) systems moving with relative velocity of magnitude $v > c$. This identity simply means that the two systems in question, N and N', are indeed superluminal to each other with $v > c$, provided the condition $m' \to -m$ is also consistently satisfied. To see this, let us write, from equation (2.13), a general form of the standard mass-velocity relation for $v > c$ as

$$m'_1 = m/r = \pm i m[1 - (v^2/c^2)]^{-1/2} \tag{2.14}$$

which agrees with the tachyon cases (Parker, 1969; Antippa, 1972; Mignani & Recami, 1973) if the -ve sign is omitted. Recall that m is a rest-mass in N, whereas m'_1 and m' (belonging to N') can be related to each other, independent of the transformation (2.3), by the familiar (subluminal, $v < c$) Einstein mass-velocity result. That is, m_1' and m' are of the same sign. Since the mass m_1' can be at rest with respect to its own inertial system N', which is superluminal to N, it follows that for the condition $m_1' = m'$ the mass m' is also at rest with respect to N'. Therefore the invariant operation such as $m_1' \to m' \to -m$ in equation (2.3) demands that the plus sign (+) in equation (2.14) should not be included. (Notice that such a guided choice of the sign remains unclear for tachyons (Parker, 1969; Antippa, 1972; Mignani & Recami, 1973) where the plus sign in the mass-velocity relation is chosen from a *priori* postulate.) Thus we see that the equation

$$m_1' = -im[1 - (v^2/c^2)]^{-1/2}, \qquad v > c \tag{2.15}$$

gives the required condition, $m_1' = m' \to -m$ for $v = (2)^{1/2}c$, confirming the consistency of the transformation (2.3). Besides, this implication is also justified from another point of view that two *identically equal masses* should remain at rest with respect to each other—a result already known from the Einstein mass-velocity relation. For a +ve sign in equation (2.14) neither this inference can be drawn nor the Dirac equation can be made invariant in our *non-trivial formalism.* Besides, the -ve sign in equation (2.15) also provides a firm basis for distinguishing an equal-valued set of energies and masses m_1', m' and m (for velocity $v = 0$ and $v = (2)^{1/2}c$ respectively) through a probable form of ± symmetry in the gravitational interaction (see footnote, p. 192). This amounts to saying that the gravitational force between m' and m is -ve (repulsive) in contrast to a +ve effect (attraction) among the same type m'- or m-particles. Although the above results could offend the traditional way of thinking, none of the physical principles is violated nor is there any justification in ignoring this new information.

3. *Conclusion*

In summary, using the first-order field equation† we have shown that symmetry principles which are Dirac- and Lorentz-invariant through equations (2.2), (2.3), and (2.11)–(2.15) lead to some interesting physics. A negative-mass-in-effect particle (referred to here as 'bisiston') emerges to be a valid candidate for our universe in the large. A m'-bisiston of charge e should move faster than the speed of light and be capable of mass-repulsion ($-G_I$) and annihilation with respect to its pair-particle (i.e., the m-electron of the same charge e). A similar positron–bisiston pair may behave in the same way. The question now is to ask: Will such particles (bisistons) ever be discovered? Where and how should we look for them? While it remains for the future to answer decisively on this issue, in our pursuit of scientific knowledge it is

† Compare with the second-order approach for tachyons (e.g. see Dhar & Sudarshan, 1968).

worthwhile including them in the list of searchable particles. The searching problem, however, does not appear to be as simple as the usual techniques of pair-particle production. Possibly, more sophisticated mechanisms and devices are required. In a subsequent paper we shall discuss a few search-proposals (for bisistons) which are currently in progress.

We conclude with the remark that gravitational symmetry (i.e., $\pm G_I$) is most likely to be of the rarest kind, hardly realisable in our laboratory (none so far), though very much probable in the wide universe. And our philosophy is comparable to that of tachyons for further exploitation.

Note added in proof: After submission of this paper the author's attention was drawn to a possible question regarding a difference between the above referred bisiston and the jugmon of a previous paper (Sinha, 1973). Indeed, bisistons and jugmons are the same particles with different names as recently shown (Sinha, 1974).

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References

Antippa, A. F. (1972). *Nuovo Cimento*, **10A**, 389.
Bilaniuk, O. M. P., Deshpande, V. K. and Sudarshan, E. C. G. (1962). *American Journal of Physics*, **30**, 718.
Bjorken, J. D. and Drell, S. D. (1964). *Relativistic Quantum Mechanics.* McGraw-Hill, New York.
Dhar, J. and Sudarshan, E. C. G. (1968). *Physical Review*, **174**, 1808; and references therein.
Dirac, P. A. M. (1930). *Proceedings of the Royal Society (London)*, **A126** 360.
Feinberg, G. (1967). *Physical Review*, **159**, 1089.
Feynman, R. P. (1962). *Quantum Electrodynamics.* W. A. Benjamin, New York.
Mignani, R. and Recami, E. (1973). *Nuovo Cimento*, **14A**, 169; and references therein.
Parker, L. (1969). *Physical Review*, **188**, 2287.
Recami, E. and Mignani, R. (1972). *Nuovo Cimento Letters*, **4**, 144.
Roman, P. (1960). *Elementary Particles.* North-Holland, Amsterdam.
Sakurai, J. J. (1958). *Nuovo Cimento*, 7, 649.
Sinha, B. B. P. (1973). *Bulletin of the American Physical Society*, **18**, 645.
Sinha, B. B. P. (1974). *Bulletin of the American Physical Society*, **19**, 1130; (1974). *New Theory of Faster-than-Light Relativity (Classical & Quantum)* . . . P. Institute of Sciences and Maths., Box 1574, Guelph, Ont., Canada Publication, Paper Nos. 1, 2, 4 (in press).
Sudarshan, E. C. G. and Marshak, R. (1958). *Physical Review*, **109**, 1860.